Using a CASIO Scientific Calculator for LCM and HCF as well as Factorisation in Grade 8 and 9

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1. Motivation

To calculate the Lowest Common Multiple (LCM) and Highest Common Factor (HCF) is not only important skills as a calculation on their own, but is used when working with fractions, not only in Grade 8 and 9 but right through to Grade 12. Factors continue to play an important role right through the curriculum.

The Scientific Calculators assist learners when they are doing numerical calculation with fractions, but without the understanding of the LCM and HCF, when learners get confronted with Algebraic Fractions, they struggle

The aim of the workshop is to use the Scientific Calculator as a tool to improve the understanding of LCM and HCF, so that learners can do the work and improve their understanding of the work. A better not only procedural understanding of the numerical concepts of factors, LCM and HCF will also increase the understanding of the algebraic concepts.

2. Introduction to the CASIO Scientific Calculator in terms of Prime Factors
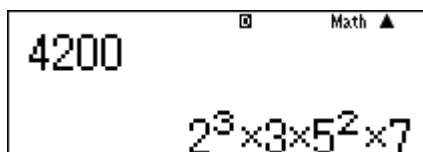
Before we even start with factors it is a good habit to always reset your calculator.
Follow the following:

$\boxed{\text{SHIFT}}$ $\boxed{9}$ $\boxed{1}$ $\boxed{=}$ $\boxed{\text{AC}}$

Prime factors can be found by using the FACT key.

If we want the prime factors of 4200, we need to enter the number in our calculator and then factorise is.

$\boxed{4}$ $\boxed{2}$ $\boxed{0}$ $\boxed{0}$ $\boxed{=}$ $\boxed{\text{SHIFT}}$ $\boxed{\circ\,{}^{\prime\prime\prime}}$

This means that
- I can write 4 200 = $2^3 \times 3 \times 5^2 \times 7$
- That 4200 has four prime factors – 2; 3; 5 and 7

1. Write the following as the product of prime factors:
    1.1. 27 440
    1.2. 5 292
    1.3. 3 600
2. List the prime factors of the following numbers:
    2.1. 24
    2.2. 1 155
    2.3. 39
3. Is 97 a prime number?
4. Is 29 a prime number?
5. Make use of the prime factors of 35 and determine, without division, if 35 is a factor of:
    5.1. $2 \times 3 \times 5 \times 7$
    5.2. $2^2 \times 3^2 \times 5$
    5.3. $2 \times 5^2 \times 7^3$
    5.4. 10×14
    5.5. 20×21
6. Determine the common prime factor(s) of the following pairs of numbers:
    6.1. 24 and 14
    6.2. 10 and 35
    6.3. 55 and 132
    6.4. 26 and 65
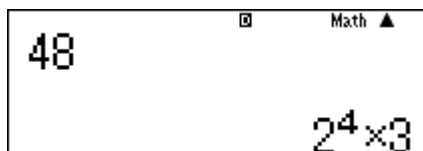   6.5. 28 and 140
    6.6. 78 and 114

3.   Finding factors

Although the calculator is programmed to give us prime factors, we can use it to assist us with far more than just the prime factors.

All factors

List all the factors of 48

$48 = 2^4 \times 3$

Now we need to engage with what this means.

Obviously 2 and 3 are both factors of 48.

But also 2^2, 2^3 and 2^4.

As well as 2×3; $2^2 \times 3$; $2^3 \times 3$ and $2^4 \times 3$.

And then of course 1 is also a factor of 48.

So how do we know that we have all the factors?

When a number is written in prime factor form, we can use the exponents of the prime factors to establish how many factors the number have. (Nice to know!) We simply add 1 to each of the exponents and multiply these numbers.

If $a = b^c \times d^e \times f^g$ .... Then a has $(c + 1)(e + 1)(g + 1)$.... factors.

$48 = 2^4 \times 3^1$ therefore 48 has $(4+1)(1+1) = 10$ factors.

When I have to find all the factors of 48, I now know that after 5 factors I can find the last 5 by division. Start at 1 and **by using the prime factors** establish the factors.

1	yes (always a factor)
2	yes
3	yes
4	yes (2^2)
5	no
6	yes (2×3)

STOP – I have my first 5

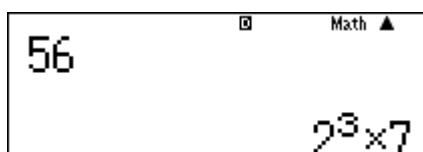|	
8	($48 \div 6$)
12	($48 \div 4$)
16	($48 \div 3$)
24	($48 \div 2$)
48	($48 \div 1$)

$F_{48} = \{1; 2; 3; 4; 6; 8; 12; 16; 24; 48\}$

Factor pairs

To write a number as the product of different "factor pairs" is a skill that really helps learners when the have to factorise trinomials in algebra. (Even when you have to find the factors of those "horrible" trinomials where the leading coefficient is not 1). What we have done with 48 is not only finding all the factors of 48, but also establishing the factor pairs.

Find all the product pairs of 56.

⌗ 5 ⌗ 6 ⌗ = ⌗ SHIFT ⌗ °᾿᾿᾿

56

Math ▲

$2^3 \times 7$

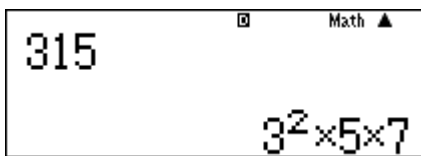So 56 has $(3 + 1)(1 + 1) = 4 \times 2 = 8$ factors and 4 factor pairs

Pairs: 1 × 56

2 × 28

4 × 14

8 × 7

(This of course means that F_{56} = {1; 2; 4; 8; 7; 14; 28; 56}

Find all the factor (product) pairs of 315

3 1 5 = SHIFT ∘ ,,,

315 has (2 + 1)(1 + 1)(1 + 1) = 3 × 2 × 2 = 12 factors and 6 factor pairs.

Pairs: 1 ×315

3 × 105

5 × 63

7 × 45

9 × 35

15 × 21

Activity 2

1. List all the factors of
   1.1. 60
   1.2. 84
   1.3. 72
   1.4. 105
2. Find all the factor pairs of
   2.1. 63
   2.2. 99
   2.3. 68
3. How many prime numbers between 20 and 50?
4. How many even prime numbers between 20 and 50?
5. Write the following as the sum of two prime numbers
   5.1. 10
   5.2. 18
   5.3. 24

Question: What is the implication if a number has an uneven number of factors?

$36 = 2^2 × 3^2$ and 36 has $(2 + 1)(2 + 1) = 9$ factors. This means 4,5 factor pairs. What does a "half pair" mean? When we start listing the pairs we find that $1 × 36$; $2 × 18$; $3 × 12$; $4 × 9$ and……. $6 × 6$.

An uneven number of factors is an indication of a perfect square and the "half pair" will then be the square root of the specific number.
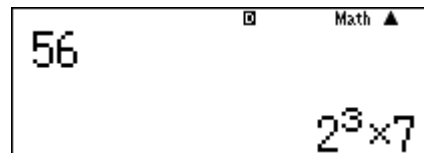
4. Highest Common Factor

We need to understand that the term "Highest Common Factor" means the biggest number that can divide into the following numbers. This is a necessity when we are simplifying fractions.

Using prime factors find the Highest Common Factor of 16 and 56

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16   2^4

56         $2^3 × 7$
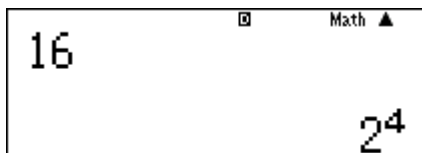
$16 = 2 × 2 × 2 × 2$

$56 = 2 × 2 × 2 × 7$

Let us find which factors are common

$16 = 2 × 2 × 2 × 2$

$56 = 2 × 2 × 2 × 7$

$HCF_{16; 56} = 2 × 2 × 2$

Activity 3

Find the HCF of:

1. 16; 56 and 72
2. 13 and 39
3. 24; 40 and 80
4. 38; 57 and 95
5. 144 and 216
6. 12; 32 and 48
7. 36 and 54
8. 35; 63 and 84
9. 24; 36 and 48
10. 25; 35 and 45
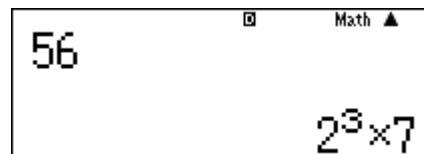
11. 36; 54 and 90

5. Lowest Common Multiple

We need to understand that the term "Lowest Common Multiple" means the smallest number that the following numbers can all divide into. This is an absolute necessity when we are adding and subtracting fractions.

Using <u>prime factors</u> find the Lowest Common Multiple of 16 and 56

| 1 | 6 | = | SHIFT | °'" |

| 5 | 6 | = | SHIFT | °'" |

	⊠	Math ▲
16		
		2^4

	⊠	Math ▲
56		
		$2^3 \times 7$
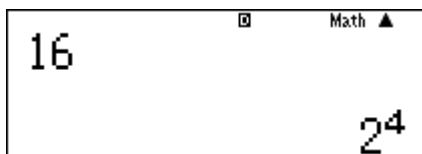
16 = 2 × 2 × 2 × 2

56 = 2 × 2 × 2 × 7

Let us find which factors are common

16 = 2 × 2 × 2 × 2

56 = 2 × 2 × 2 × 7

These three factors do not have to be repeated in the LCM

16 = 2 × 2 × 2 × 2

56 = 2 × 2 × 2 × 7

These factors are not common.

LCM$_{16; 56}$ = 2 × 2 × 2 × 2 × 7

We can also find the LCM of more than two numbers.

Find the LCM of 16, 28 and 56.

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| 5 | 6 | = | SHIFT | °'" |
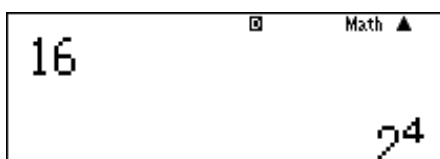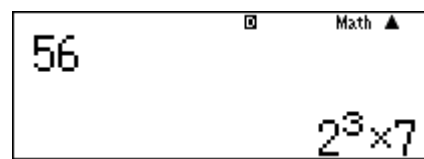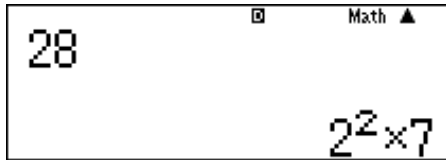
	⊠	Math ▲
16		
		2^4

	⊠	Math ▲
56		
		$2^3 \times 7$

| 2 | 8 | = | SHIFT | °'" |

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                                    D        Math ▲
28


                                       2²×7
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$16 = 2 × 2 × 2 × \textcircled{2}$

$28 = 2 × 2 × 7$

$56 = 2 × 2 × 2 × 7$

LCM = 2 × 2 × 2 × 7 × 2 = 112

From this example we should see that even where a factor is shared by just two of the numbers we will list it only once.
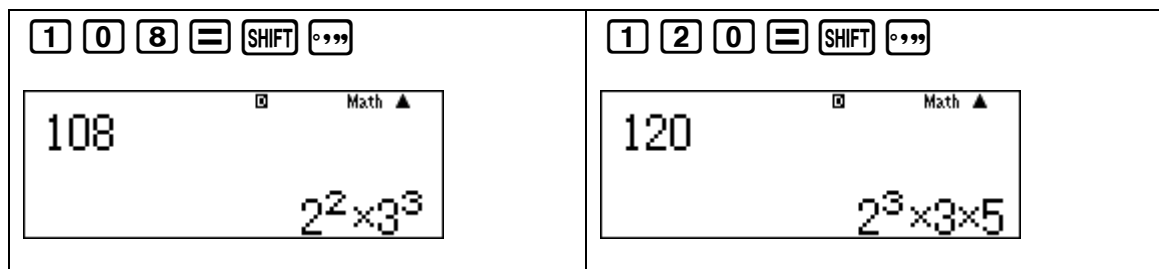
Activity 4
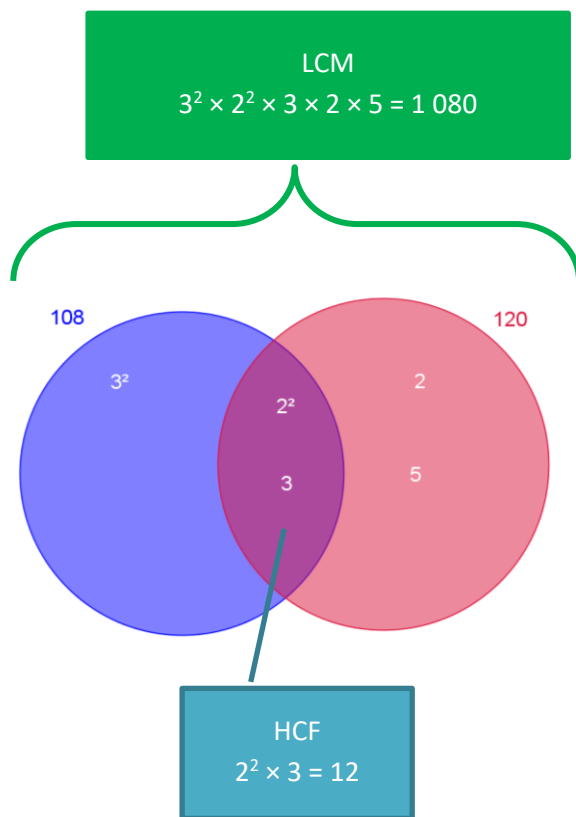
Find the LCM of:

1. 16; 56 and 72
2. 13 and 39
3. 24; 40 and 80
4. 38; 57 and 95
5. 12; 32 and 48
6. 36 and 54
7. 35; 63 and 84
8. 24; 36 and 48
9. 25; 35 and 45

6. An interesting look at HCF and LCM

Consider the numbers 108 and 120.


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1 0 8 = SHIFT ° ’ ’’            1 2 0 = SHIFT ° ’ ’’
                   D    Math ▲                     D    Math ▲
108                            120


             2²×3³                          2³×3×5
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Organising the factors with the use of a Venn diagram will result into this:

7.   How can my prime factor key help when I am doing Algebra?

Although this is a workshop on its own, I would like to spend a few minutes on factorising trinomials.

If we look at the following trinomials

$x^2 + 57x + 56$
$x^2 + 30x + 56$
$x^2 + 28x + 56$
$x^2 + 15x + 56$
$x^2 - 57x + 56$
$x^2 - 30x + 56$
$x^2 - 28x + 56$
$x^2 - 15x + 56$
$x^2 + 55x - 56$
$x^2 + 26x - 56$
$x^2 + 10x - 56$
$x^2 + x - 56$
$x^2 - 55x - 56$
$x^2 - 26x - 56$
$x^2 - 10x - 56$
$x^2 - x - 56$

We need to choose the correct "factor pair" of 56 to find the two factors of the trinomial.

Let's consider our earlier activity

So $56 = 1 \times 2 \times 2 \times 2 \times 7$

Pairs: 1×56
    2×28
    4×14
    8×7

1 and 56: $x^2 + 57x + 56 = (x + 1)(x + 56)$

$x^2 - 57x + 56 = (x - 1)(x - 56)$

$x^2 + 55x - 56 = (x - 1)(x + 56)$

$x^2 - 55x - 56 = (x + 1)(x - 56)$

Make up your own trinomials for the rest of the pairs