Using a scientific calculator for factorisation of trinomials in Grade 10

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CASIO calculators

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1. Motivation

Factorising of trinomials has been part of every Mathematics curriculum in the world and a lot of learners struggle with it – especially when the leading coefficient is not 1.

The aim of the workshop is to use the Scientific Calculator as a tool to improve the understanding of factor pairs, so that learners can do the work and improve their understanding of the work. Thereafter we are going to look at a possible alternative method for factorising trinomials

2. Planning of Workshop

10 minutes:	Introduction to the Scientific Calculator in terms of prime factors.
10 minutes:	Finding factor pairs by using prime numbers
10 minutes:	Trinomials with leading coefficient 1
30 minutes:	Trinomials with leading coefficient not 1

3. Introduction to the CASIO Scientific Calculator in terms of Prime Factors

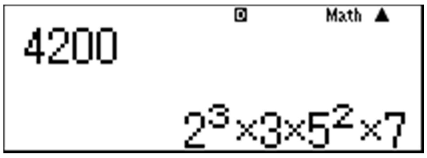
Before we even start with factors it is a good habit to always reset your calculator.
Follow the following:

[SHIFT] [9] [1] [=] [AC]

Prime factors can be found by using the FACT key.

If we want the prime factors of 4200, we need to enter the number in our calculator and then factorise is.

[4] [2] [0] [0] [=] [SHIFT] [°'"]



This means that

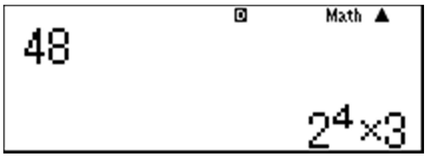
- I can write 4 200 = $2^3 \times 3 \times 5^2 \times 7$
- That 4200 has four prime factors – 2; 3; 5 and 7

4. Finding factors

Although the calculator is programmed to give us prime factors, we can use it to assist us with far more than just the prime factors.

All factors

List all the factors of 48



48 = $2^4 \times 3$

Now we need to engage with what this means.

Obviously 2 and 3 are both factors of 48.

But also 2^2, 2^3 and 2^4.

As well as 2×3; $2^2 \times 3$; $2^3 \times 3$ and $2^4 \times 3$.

And then of course 1 is also a factor of 48.

So how do we know that we have all the factors?

When a number is written in prime factor form, we can use the exponents of the prime factors to establish how many factors the number have. (Nice to know!) We simply add 1 to each of the exponents and multiply these numbers.

If $a = b^c \times d^e \times f^g$ …. Then a has $(c + 1)(e + 1)(g + 1)$…. factors.

48 = $2^4 \times 3^1$ therefore 48 has $(4+1)(1+1) = 10$ factors.

When I have to find all the factors of 48, I now know that after 5 factors I can find the last 5 by division. Start at 1 and **by using the prime factors** establish the factors.

1	yes (always a factor)
2	yes
3	yes
4	yes (2^2)
5	no
6	yes (2×3)

STOP – I have my first 5

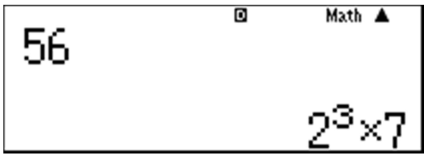
8 (48 ÷6)
12 (48 ÷4)
16 (48 ÷3)
24 (48 ÷2)
48 (48 ÷1)

F_{48} = {1; 2; 3; 4; 6; 8; 12; 16; 24; 48}

Factor pairs

To write a number as the product of different "factor pairs" is a skill that really helps learners when the have to factorise trinomials in algebra

Find all the product pairs of 56.



So 56 has (3 + 1)(1 + 1) = 4 × 2 = 8 factors and 4 factor pairs

Pairs: 1 × 56

2 × 28

4 × 14

8 × 7

(This of course means that F_{56} = {1; 2; 4; 8; 7; 14; 28; 56}

Activity 1

1. List all the factors pairs of
   1.1. 60
   1.2. 84
   1.3. 72
   1.4. 105
   1.5. 63
   1.6. 99
   1.7. 68
   1.8. 24
2. List all factor pairs of
   2.1. 36
   2.2. 16
   2.3. 100

Question: What is the implication if a number has an uneven number of factors?

36 = $2^2 \times 3^2$ and 36 has (2 + 1)(2 + 1) = 9 factors. This means 4,5 factor pairs. What does a "half pair" mean? We realise that when we start listing the pairs we find that 1 × 36; 2 × 18; 3 × 12; 4 × 9 and……. 6 × 6.

An uneven number of factors are an indication of a perfect square and the "half pair" will then be the square root of the specific number.

5. Trinomials with leading coefficient 1

Consider the product of the following binomials: $(x+a)(x+b)$

$$\begin{aligned}(x+a)(x+b) &= x^2+ax+bx+ab \\ &= x^2+(a+b)x+ab\end{aligned}$$

The constant value is ab the **product** of two numbers and the coefficient of the middle term is the **sum** of the same to numbers.

$x^2 + 11x + 24$

The trinomial $x^2 + 11x + 24$ should therefore have two binomial factors of the form $(x + a)(x + b)$ where $ab = 24$ and $a + b = 11$

We need the factor pairs of 24:

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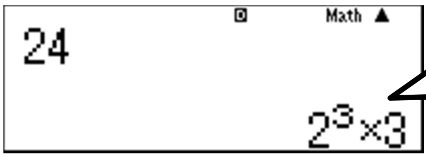


[(3+1) × (1+1)] ÷ 2
= 4 factor pairs

Factor pairs:

1 × 24
2 × 12
3 × 8
4 × 6

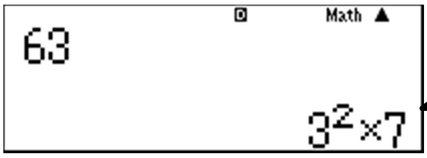
Sum of number pairs:

1 + 24 = 25
2 + 12 = 14
3 + 8 = 11
4 + 6 = 10

$$x^2+11x+24=(x+3)(x+8)$$

$x^2 - 16x + 63$

The trinomial $x^2 - 16x + 63$ should therefore have two binomial factors of the form $(x + a)(x + b)$ where $ab = 63$ and $a + b = -16$ ….. which actually implies that the binomials is of the form $(x - a)(x - b)$ [the product of two negative numbers is a positive number and the sum of two negative numbers is a negative number].



[(2+1) × (1+1)] ÷ 2
= 3 factor pairs

Factor pairs:

1 × 63
3 × 21
9 × 7

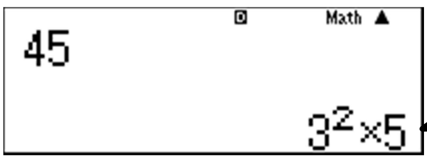
Sum of factor pairs (remember that we have to work with two negative numbers….)

–1 – 63
–3 – 21
–9 – 7

$x^2 - 16x + 63 = (x - 9)(x - 7)$

$x^2 - 12x - 45$

The trinomial $x^2 - 12x - 45$ should therefore have two binomial factors of the form $(x + a)(x + b)$ where $ab = -45$ and $a + b = -12$ ….which actually implies that the binomials is of the form $(x - a)(x + b)$ [the product of a positive and a negative number is a negative number].



[(2+1) × (1+1)] ÷ 2
= 3 number pairs

1 × 45
3 × 15
9 × 5

Sum of factor pairs (remember that we have to work with a positive and a negative number….)

–1 + 45 or 1 – 45
–3 + 15 or 3 – 15
–9 + 5 or 9 – 5

$x^2 - 16x + 63 = (x + 3)(x - 15)$

Activity 2

Factorise the following:

1. $x^2+9x+14$
2. $x^2+9x+18$
3. $x^2+7x+10$
4. $x^2+12x+35$
5. $x^2+252x+500$
6. $x^2+18x+32$
7. $x^2-10x+25$
8. $x^2-13x+30$
9. $x^2-15x+26$
10. $y^2-9y+20$
11. $x^2-15x+26$
12. $(3x^2-21x+36)(2x^2+10x+12)$
13. x^2-x-6
14. x^2+5x-6
15. $x^2-4x-21$
16. $x^2+7x-18$
17. $pq^2+9pq+18p$
18. $2x^2-2x-24$
19. $(7x^2-22x+3)(2x^2+9x+10)$
20. $(6x^2-5x+1)(2x^2+9x+9)$

And here is a fun exercise to do

1. $x^2+57x+56$
2. $x^2+30x+56$
3. $x^2+28x+56$
4. $x^2+15x+56$
5. $x^2-57x+56$
6. $x^2-30x+56$
7. $x^2-28x+56$
8. $x^2-15x+56$
9. $x^2+55x-56$
10. $x^2+26x-56$
11. $x^2+10x-56$
12. x^2+x-56
13. $x^2-55x-56$
14. $x^2-26x-56$
15. $x^2-10x-56$
16. x^2-x-56

6. Trinomials where the leading coefficient is not 1

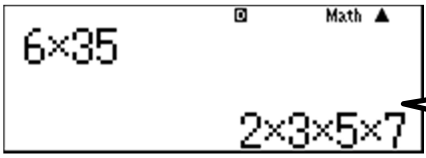
Consider the product of the following binomials: $(px+q)(rx+s)$

$$\begin{aligned}(px+q)(rx+s) &= prx^2 + psx + qrx + qs \\ &= prx^2 + (ps+qr)x + qs\end{aligned}$$

Looking at the constant and comparing it with the coefficient of x does not assist us. However, if we multiply the leading coefficient with the constant we get *prqs* which can be written as *ps* × *qs*. Then the sum of these two factors will be the coefficient of x.

$6x^2 + 29x + 35$

The trinomial $6x^2 + 29x + 35$ should therefore have two binomial factors of the form $(px + q)(rx + s)$ where $pqrs = 6 \times 35 = 210$ and $ps + qr = 29$



[(1+1) × (1+1)× (1+1) × (1+1)] ÷ 2
= 8 factor pairs

Factor pairs:

1 × 210
2 × 105
3 × 70
5 × 42
6 × 35
7 × 30
10 × 21
14 × 15

Sum of Factor pairs:

1 + 210 = 211
2 + 105 = 107
3 + 70 = 73
5 + 42 = 47
6 + 35 = 41
7 + 30 = 37
10 + 21 = 21
14 + 15 = 29

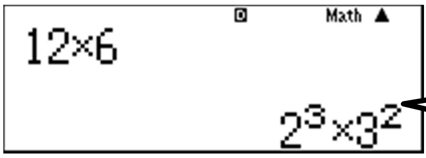
From here there is a slight deviation from the factorising of a trinomial where the leading coefficient is 1

$$\begin{aligned}6x^2 + 29x + 35 &= 6x^2 + 14x + 15x + 35 \\ &= 2x(3x+7) + 5(3x+7) \\ &= (3x+7)(2x+5)\end{aligned}$$

$12x^2 - 17x + 6$

The trinomial $12x^2 - 17x + 6$ should therefore have two binomial factors of the form $(px + q)(rx + s)$ where $pqrs = 12 \times 6 = 72$ and $ps + qr = -17$ [so actually $(px - q)(rx - s)$]

1 2 × 6 = SHIFT °’’’



[(3+1) × (2+1)] ÷ 2
= 6 factor pairs

Factor pairs:

- 1 × 72
- 2 × 36
- 3 × 24
- 4 × 18
- 6 × 12
- 8 × 9

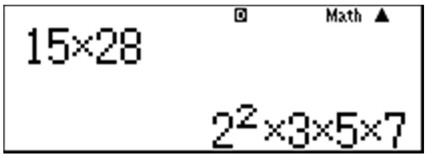
Sum of Factor pairs:

- –1 – 72 = –73
- –2 – 36 = –387
- –3 – 24 = –27
- –4 – 18 = –22
- –6 – 12 = –18
- –8 – 9 = –17

$$\begin{aligned} 12x^2 - 17x + 6 &= 12x^2 - 8x - 9x + 6 \\ &= 4x(3x - 2) - 3(3x - 2) \\ &= (3x - 2)(4x - 3) \end{aligned}$$

$15x^2 + x - 28$

The trinomial $15x^2 + x - 28$ should therefore have two binomial factors of the form $(px + q)(rx + s)$ where $pqrs = 15 \times -28 = -420$ and $ps + qr = 1$ [so actually $(px - q)(rx + s)$]



[(2+1) × (1+1) × (1+1) × (1+1)] ÷ 2
= 12 factor pairs

Factor pairs:

1 × 420
2 × 210
3 × 140
4 × 105
6 × 70
7 × 60
10 × 42
12 × 35
15 × 28
20 × 21 $\quad -20 + 21 = 1$

$$
\begin{aligned}
15x^2 + x - 28 &= 15x^2 - 20x + 21x - 28 \\
&= 5x(3x - 4) + 7(3x - 4) \\
&= (3x - 4)(5x + 7)
\end{aligned}
$$

Activity 3

Fully factorise the following trinomials:

1. $8x^2 + 10x + 3$
2. $6x^2 + 11x + 3$
3. $2x^2 + 7x + 5$
4. $6x^2 + 7x + 2$
5. $7x^2 + 23x + 6$
6. $10x^2 + 9x + 2$
7. $15x^2 + 14x + 3$
8. $6x^2 + 13x + 5$
9. $4x^2 + 27x + 18$
10. $4x^2 + 31x + 21$
11. $9x^2 + 6x + 1$
12. $10x^2 + 29x + 10$
13. $2x^2 + 5x + 3$
14. $4x^2 + 25x + 6$

15. $16x^2+8x+1$
16. $25x^2+40x+16$
17. $36x^2+60x+25$
18. $21x^2+20x+4$
19. $6x^2+9x+3$
20. $12x^2+18x+6$
21. $6x^2+10x+4$
22. $4x^3+10x^2+6x$
23. $36x^3+42x^2+12x$
24. $20x^3+48x^2+16x$
25. $12x^3y+26x^2y^2+12xy^3$
26. $72x^3+192x^2y+12xy^2$
27. $9x^4y^2+15x^3y^3+6x^2y^4$
28. $6x^2-7x+2$
29. $3p^2-10p+8$
30. $6n^2-13n+6$
31. $3x^2-23x+14$
32. $x^2-10x+25$
33. $6x^2-19x+10$
34. $x^2-13x+30$
35. $2x^2-28x+90$
36. $35-12x+x^2$
37. $x^2-18x+80$
38. $y^2-9y+20$
39. $x^2-15x+26$
40. $8x^2-28x+12$
41. $15x^2-16x+4$
42. $2x^2-6x+10$
43. $4x^2-48x+144$
44. $10x^2-19x+6$
45. $6x^2-17x+12$
46. $12x^2-34x+24$
47. $10x^2-17x+3$
48. $(6x^2-5x+1)(2x^2+9x+9)$
49. $(7x^2-22x+3)(2x^2+9x+10)$